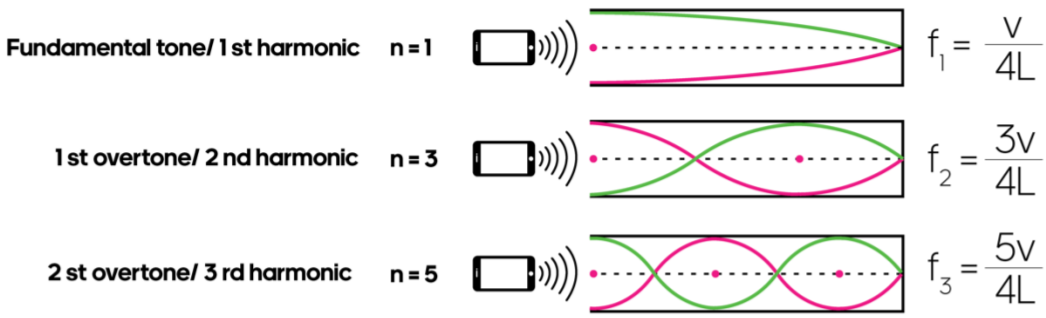
Study of Standing Waves to Find Speed of Sound in Air



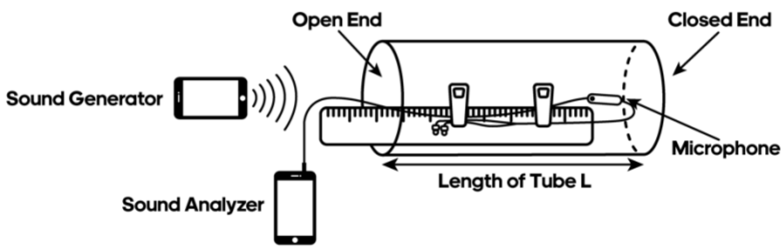


Fig 1

Purpose

Using mobile devices as sound analyzer and sound generator to study standing waves and determine the speed of sound in air.

Theory

- The velocity of a traveling wave v is derived by dividing one wavelength λ (unit in m) with one period *T* (time taken for the wave to travel one wavelength, unit in s). The reciprocal of period is frequency f (unit in Hz). The wave formula is therefore given by

$$v = \frac{\lambda}{T} = f\lambda \text{..........(1)}$$

  where v = velocity of wave; f = frequency of wave; λ = wavelength of wave.
- Waves share some common properties – **interference**, **reflection**, **refraction** and **diffraction**. In this experiment, the properties of interference and reflection will be utilized to find the speed of sound.
- **Standing wave**, also called **stationary wave**, is a wave in a medium in which every point on the axis of the wave propagation has its corresponding constant amplitude, i.e. certain points (called "nodes") give zero resultant amplitude, certain points (called "antinodes") oscillate between maximum amplitude and other points oscillate at fixed amplitudes depending on their positions. The wave appears "standing" or "stationary", not traveling in either direction. It provides an easier way to study the speed of sound.

- Standing wave can be generated by two waves traveling in opposite directions and superposing with one another at appropriate **resonant frequencies** such that **resonance** occurs. Resonance gives the loudest sound as the waves constructively interfere with each other and constitute maximum amplitude at the antinodes. (Fig 7 in Appendix)
- A one-end closed glass tube is used to simplify the experiment. When a sound wave is generated at the open end of the tube, it will propagate inside the tube and be reflected about x-axis by the closed solid end of the tube, generating a wave traveling backward with identical frequency.
- To understand this condition, let us go through some mathematical expressions of the waves. Given the generated sound wave is $y_1 = A \sin(kx - \omega t)$, where A is the amplitude, k is wave number where $k = \frac{2\pi}{\lambda}$, ω is the angular frequency where $\omega = \frac{2\pi}{T}$ or $2\pi f$. The reflected wave will be given by $y_2 = A \sin(kx + \omega t)$, where A, k, ω are the same for y_1 and y_2 at all position x and time t.
- By the principle of superposition, the resultant wave of y_1 and y_2 is given by

  $$y = y_1 + y_2 = A \sin(kx - \omega t) + A \sin(kx + \omega t) \text{..........(2)}$$

  Using the trigonometric sum-to-product identity $\sin\alpha \pm \sin\beta \equiv 2\cos\left(\frac{\alpha \mp \beta}{2}\right)\sin\left(\frac{\alpha \pm \beta}{2}\right)$ and the fact that $\cos(-\alpha) = \cos\alpha$,

  $$y = 2A\cos(\omega t)\sin(kx) \text{..........(3)}$$

- At any time t, to find the nodes ($y = 0$), $\sin(kx)$ has to be 0. For $\sin\frac{2\pi x}{\lambda} = 0$, $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots$ The antinodes are when $\sin(kx)$ equals 1, i.e. $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$ The interval between consecutive nodes and antinodes is one-fourth the wavelength $\frac{\lambda}{4}$.
- To fulfill the boundary condition in the one-end closed tube, the wavelengths can be determined by $\lambda = \frac{4L}{n}$ and by $v = f\lambda$, the resonant frequency can be obtained from

  $$f = \frac{nv}{4L} \text{..........(4)}$$

  where L = Length of the tube, n = positive odd integer (1, 3, 5, 7…) (As shown in Fig 1)
- Increasing from low frequency, the first resonance is called the "fundamental tone" or "1^{st} harmonic", the second resonance is called the "1^{st} overtone" or "2^{nd} harmonic" and so on.
- In this experiment, the relationship between length of tube and resonance frequencies will be testified. The positions of nodes and antinodes and thus the wavelengths of resonance will be found. The speed of sound can be deduced using the wave formula. Its theoretical

value is given by $340 ms^{-1}$.

Apparatus

- 2 mobile devices with "AP-Sensor" app
- A one-end closed glass tube
- A ruler
- An earphone with microphone and 3.5mm audio jack
- A clip for fixing the microphone

Setup

Caution: Fix the glass tube with fixer to avoid breaking the glass tube and cause injury.

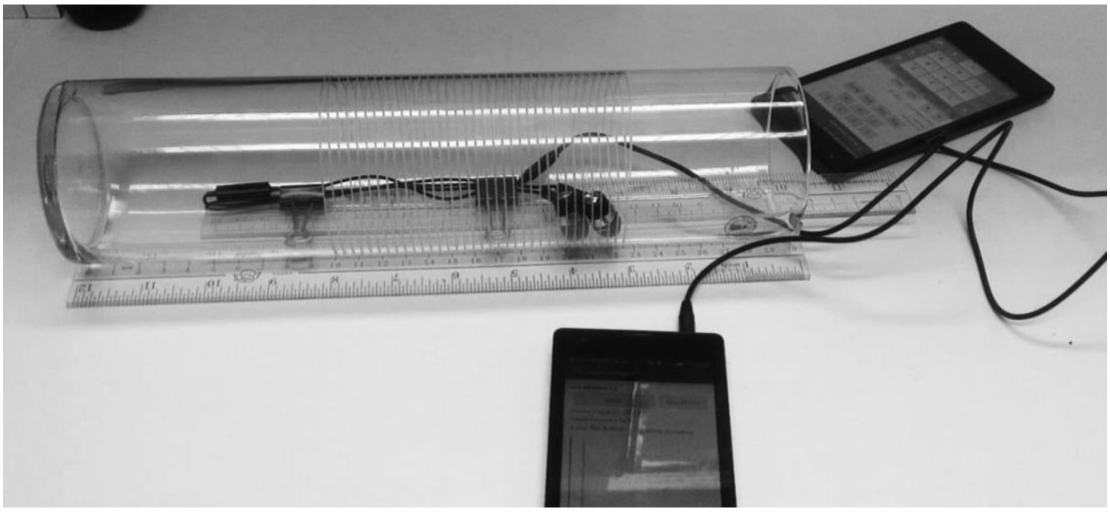


Fig 2

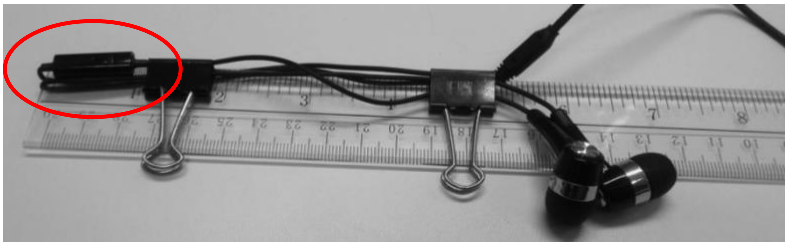


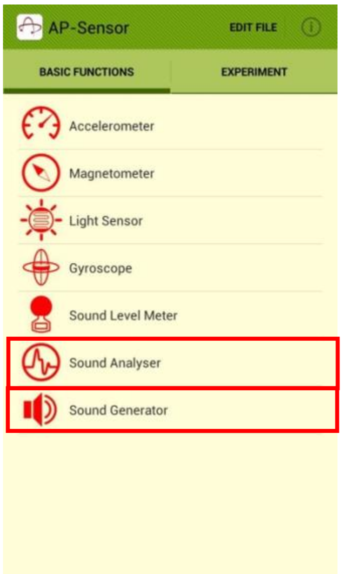
Fig 3

Procedure

Set up the Experiment

1. Measure and record the length of the glass tube, neglecting the thickness of the wall. Estimate in Table 1 the resonant frequencies for the first 5 harmonics using equation (4).
2. Clamp the earphone on the ruler such that the microphone is at the end of the ruler (Fig 3).
3. Connect the earphone to one of the mobile device (Sound analyzer). Put the microphone clamped on the ruler into the glass tube. Place another mobile device (Sound generator) where its speaker points towards the open end of the tube (Fig 1 & 2).
4. Run the app “AP-Sensor” on both mobile devices. Start “Sound Generator” and “Sound Analyzer” in corresponding device under “Basic Functions” (Fig 4).

Fig 4



Find Resonant Frequencies (Fig 5)

5. In the sound generator, enter the estimated 1st harmonic frequency in the field “Enter Frequency”.
6. Press “Gen” to generate the sound wave at that frequency.
7. Point the speaker of the device to the open end of the tube.
8. Finely adjust the frequency up or down by 1Hz each time pressing the “Up/Down arrow” button to find resonant frequency of 1st harmonic. When the resonant frequency is reached, the sound will be significantly louder.
9. Record the experimental resonant frequency in Table 1.

Fig 5

Find Positions of Nodes and Antinodes

10. In the sound analyzer, press the button “Waveform” to display waveform of detected wave. Sweep the graph with two fingers to zoom the graph to a scale suitable for observation (Fig 6).
11. Move the microphone from the closed end to the open end along the centre of the tube. At nodes, the amplitude of waveform will be minimum. At antinodes, the amplitude of waveform will be maximum. Using the ruler, record all the positions of the nodes and antinodes from the closed end of glass tube in Table 2.
12. Determine the wavelengths by taking average distance between adjacent nodes and antinodes and calculate the speed of sound in Table 3.

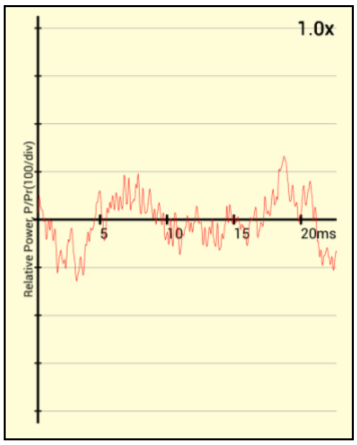


Fig 6

13. Repeat Step 5 to 12 with higher (2nd to 5th) harmonics by increasing the generated sound to higher resonant frequencies. (Optional: Repeat the experiment with 6th or above harmonics.)
14. Optional: Repeat the experiment with glass tube of different length to observe the difference in resonant frequencies.

Data

Length of the tube L: _______________m

Table 1: Estimated resonant frequencies for different harmonics (assume speed of sound in air v = $340 ms^{-1}$)

Harmonic	n	Theoretical Resonant Frequency (Hz) $\boldsymbol{f = \frac{nv}{4L}}$	Experimental Resonant Frequency (Hz)
1st harmonic	1		
2nd harmonic	3		
3rd harmonic	5		
4th harmonic	7		
5th harmonic	9		

Table 2: Positions of nodes and antinodes for different harmonics

Harmonic	Position(s) of Antinode(s) from Closed End of the Tube (m)					Position(s) of Node(s) from Closed End of the Tube(m)					
1st harmonic						0					L =
2nd harmonic						0					L =
3rd harmonic						0					L =
4th harmonic						0					L =
5th harmonic						0					L =

Table 3: Wavelength and speed of sound wave

Harmonic	Average Node & Antinode Separation (m)	Wavelength (m) $\boldsymbol{\lambda}$ 4x(Average Node & Antinode Separation)	Speed of Sound (ms^{-1}) $\boldsymbol{v = f\lambda}$
1st harmonic			
2nd harmonic			
3rd harmonic			
4th harmonic			
5th harmonic			

Discussion

1. How does the loudness of sound vary when the frequency is gradually adjusted to resonant frequency?
2. In Table 1, how do the resonant frequencies found in the experiment differ from the theoretical value determined by equation (4)? If the discrepancies are large, what would be the possible reasons?
3. In Table 2, how is the precision of separation between adjacent nodes/antinodes for different harmonics in the experiment, compared to the illustration in Fig 1?
4. In Table 3, there are other possible ways to determine the wavelength of wave, such as doubling separation between adjacent nodes or that between adjacent antinodes. Will there be any significance using different approaches?
5. How are the experimental values of speed of sound in air obtained in Table 3 compared to the literature value ($340ms^{-1}$ for HKDSE syllabus)? What are the possible errors or limitations of the experiment?
6. If 6^{th} harmonics or above are studied, what difficulty will be encountered?
7. If other glass tubes with different lengths are used to repeat the experiment, are the results of resonant frequencies and speed of sound consistent with the theory?

Appendix Fig 7